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Improvement of Analytical Model Using Uncertain Test Data

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The localization and correction of errors in computational models are discussed. A parametric optimization technique, based on the sensitivity analysis of measured eigensolutions and with regard to stiffness and mass parameters of a finite element model, was used. Uncertain eigensolutions are considered in the localization process due to fuzziness present in the experimental data used. To manipulate the uncertainties in the localization process, a fuzzy model is introduced and discussed. A numerical example is considered to illustrate the computational approach. The impact of the uncertain eigensolutions on the localization process is assessed by comparing the present numerical results with those given by the sensitivity-based method.

Nomenclature

$\boldsymbol{b}$	= distance between the structure and its finite element model, $\in R^{m(c+1),1}$
$\tilde{\boldsymbol{b}}$	= vector of errors, fuzzy, $\in R^{m(c+1),2}$
c	= number of measured degrees of freedom (DOF)
$M^{(m)}$, $K^{(m)}$	= mass and stiffness matrices of the finite element estimation, $\in R^{n,n}$
$M^{(s)}$, $K^{(s)}$	= mass and stiffness matrices of the real structure, $\in R^{n,n}$
m	= number of measured eigenmodes
m_i, k_i	= localization indicators for the ith macroelement
(m)	= quantities related to the initial estimation
n	= number of model DOF
r	= number of subdomains introduced for localization
S	= sensitivity matrix, $\in R^{m(c+1),2r}$
(s)	= quantities related to the structure
$z_\nu^{(m)}$	= νth calculated eigenvector corresponding to the c observed DOF, $\in R^{c,1}$
$z_\nu^{(s)}$	= νth identified eigenvector corresponding to the c observed DOF, $\in R^{c,1}$
ΔM, ΔK	= mass and stiffness matrices of correction, $\in R^{n,n}$
$\Delta \boldsymbol{p}$	= vector of physical correction parameters, $\in R^{2r,1}$
$\Delta \tilde{\boldsymbol{p}}$	= vector of physical correction parameters, fuzzy, $\in R^{2r,2}$
$\lambda_\nu^{(m)}$	= νth calculated eigenvalue
$\lambda_\nu^{(s)}$	= νth identified eigenvalue

Introduction

WHEN elaborating a finite element model, many problems can arise. Some of them concern the validity of the elaborated model. Many sources of errors, identified or not, can lead to the construction of a representative model that is more or less erroneous. Therefore, model validation proves necessary.

This is generally obtained by a correlation of errors between the analytical prediction and the measured behavior. The latter is then exploited to adjust the physical parameters of the model. For many years, significant effort has been made to find efficient solutions to model updating problems.

Many adjustment strategies have been proposed. Most of them use the identified modal characteristics by exploiting the orthogonality relations,[1] sensitivity techniques,[2] or concept of error in the constitutive equation.[3] Another type of method is based on the minimization of the errors on the transfer functions.[4]

In reality, it is impractical to adjust all of the model subdomains because such a situation leads to a large model and enormous numerical conditioning problems. Therefore, it is necessary to locate first the subdomains presenting dominant modeling errors on which corrections are performed in priority.

Therefore, in practice, updating procedures are based on two complementary phases, which are 1) localization of dominant modeling errors and 2) correction of the localized subdomains.

By hypothesis, the measured eigensolutions are a reference in all of the ways of improving the dynamic behavior of finite element models. Thus, in the existing approaches, the error is imputed to the analytical model. Nevertheless, many authors have made an inventory of errors that arise at the experimental stage. They show clearly that the preceding hypothesis is often erroneous. For example, Berman and Nagy[5] propose a classification based on two categories of errors. The first category concerns errors linked to poor measures. The second category groups all of the errors linked to a bad correlation between the structure test and its finite element model. Therefore, it can be assumed that the experimental modal characteristics exploited in an updating process are imprecise.

Some approaches, such as Monte Carlo simulations or the stochastic method, deal with imprecision. A new approach, based on both an existing updating method and a fuzzy formalism, is proposed. In our case, the imprecision on the experimental modal characteristics is taken into account by using fuzzy formalism. Unlike the probabilistic techniques, this method deals with both imprecision and uncertainties.

The local updating sensitivity method developed in the Applied Mechanics Laboratory, Besançon, France, has been chosen.[6] The imprecision modeling, when it is used with this updating sensitivity method, leads to the resolution of a fuzzy linear system with real coefficients. The solution of such a system has already been applied successfully in the case of imprecise boundaries conditions analysis.[7]

After briefly outlining the principles of the traditional updating sensitivity method, a modification is proposed that takes the error due to the experiments into account.

Finally, the performance of the proposed approach is evaluated and compared with an application to a numerical simulation test case.

Localization

Definition of the Parameters

Because the general description of this method can be found in several references, e.g., Ref. 8, only its fundamental characteristics are presented.

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The relation between the matrices $M^{(m)}$ and $K^{(m)}$ of the initial estimation and the matrices $M^{(s)}$ and $K^{(s)}$ belonging to the real structure are given by

$$M^{(s)} = M^{(m)} + \Delta M, \qquad K^{(s)} = K^{(m)} + \Delta K \tag{1}$$

The affinity hypothesis for subdomains expresses the correction matrices ΔK and ΔM by linear combinations of the matrices associated to the r subdomains

$$\Delta M = \sum_{i=1}^{r} m_i M_i^{(m)}, \qquad \Delta K = \sum_{i=1}^{r} k_i K_i^{(m)} \tag{2}$$

where the quantities k_i and m_i are closely connected to the element stiffness $K_i^{(m)}$ and element mass $M_i^{(m)}$ matrices, respectively.

When the first-order terms of the Taylor expansion are taken into account, the relation between eigenvalues $\lambda_\nu^{(m)}$ and eigenvectors $z_\nu^{(m)}$ of the analytical model and eigenvalues $\lambda_\nu^{(s)}$ and eigenvectors $z_\nu^{(s)}$ of the real structure can be written for the eigenvectors as

$$z_\nu^{(s)} = z_\nu^{(m)} + \sum_{i=1}^{r} \frac{\partial z_\nu^{(m)}}{\partial k_i} \Delta k_i + \sum_{i=1}^{r} \frac{\partial z_\nu^{(m)}}{\partial m_i} \Delta m_i + \mathcal{O}(\Delta^2 k_i) + \mathcal{O}(\Delta^2 m_i) \tag{3}$$

and for the eigenvalues as

$$\lambda_\nu^{(s)} = \lambda_\nu^{(m)} + \sum_{i=1}^{r} \frac{\partial \lambda_\nu^{(m)}}{\partial k_i} \Delta k_i + \sum_{i=1}^{r} \frac{\partial \lambda_\nu^{(m)}}{\partial m_i} \Delta m_i + \mathcal{O}(\Delta^2 k_i) + \mathcal{O}(\Delta^2 m_i) \tag{4}$$

where $(\nu = 1, 2, \ldots, m)$ and m is the number of selected eigenvalues and eigenvectors for the sensitivity method. The changes of the parameters k_i and m_i are described by Δk_i and Δm_i.

Regrouping the system of linear algebraic equations (3) and (4), the system can be written in matrix form

$$S \Delta \boldsymbol{p} = \boldsymbol{b} \tag{5}$$

where

$$\boldsymbol{b}^T = \Big[\big(z_1^{(s)} - z_1^{(m)}\big)^T, \ldots, \big(z_m^{(s)} - z_m^{(m)}\big)^T, \big(\lambda_1^{(s)} - \lambda_1^{(m)}\big), \ldots, \big(\lambda_m^{(s)} - \lambda_m^{(m)}\big)\Big] \qquad [(c+1) \times m] \tag{6}$$

$$\Delta \boldsymbol{p}^T = [\Delta k_1, \ldots, \Delta k_r, \Delta m_1, \ldots, \Delta m_r] \qquad \{2 \times r\} \tag{7}$$

$$S = \begin{bmatrix} \frac{\partial z_1^{(m)}}{\partial k_1} & \cdots & \frac{\partial z_1^{(m)}}{\partial k_r} & \frac{\partial z_1^{(m)}}{\partial m_1} & \cdots & \frac{\partial z_1^{(m)}}{\partial m_r} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial z_m^{(m)}}{\partial k_1} & \cdots & \frac{\partial z_m^{(m)}}{\partial k_r} & \frac{\partial z_m^{(m)}}{\partial m_1} & \cdots & \frac{\partial z_m^{(m)}}{\partial m_r} \\ \frac{\partial \lambda_1^{(m)}}{\partial k_1} & \cdots & \frac{\partial \lambda_1^{(m)}}{\partial k_r} & \frac{\partial \lambda_1^{(m)}}{\partial m_1} & \cdots & \frac{\partial \lambda_1^{(m)}}{\partial m_r} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \lambda_m^{(m)}}{\partial k_1} & \cdots & \frac{\partial \lambda_m^{(m)}}{\partial k_r} & \frac{\partial \lambda_m^{(m)}}{\partial m_1} & \cdots & \frac{\partial \lambda_m^{(m)}}{\partial m_r} \end{bmatrix} \qquad [(c+1) \times m; 2 \times r] \tag{8}$$

with $(c + 1) \times m > 2 \times r$ and $c \gg m$.

Note $\boldsymbol{b}$ is the distance vector between the calculated and identified eigensolutions. The element in S can be obtained by deriving the eigenvalue equation and the orthogonality condition

$$\big(K^{(m)} - \lambda_\nu^{(m)} M^{(m)}\big) = 0, \qquad {}^T z_\nu^{(m)} M^{(m)} z_\nu^{(m)} = I \tag{9}$$

A method of obtaining these derivatives is presented in Ref. 9.

It must be remembered that for this localization method, the error is attributed to the analytical model. The vector $\boldsymbol{b}$ is, therefore, entirely defined by the distance that forms the analytical eigensolutions with regard to those measured on the prototype.

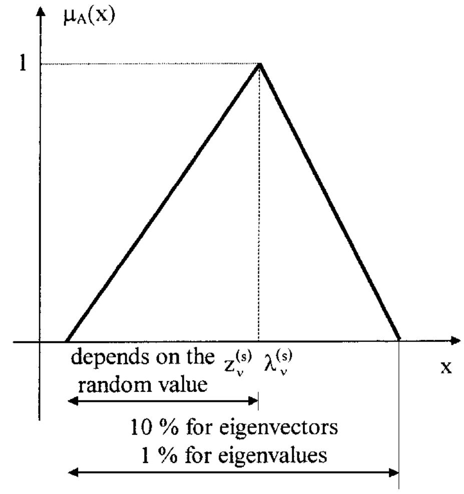

Fig. 1 Modeling of the imprecision on experimental eigensolutions by triangular fuzzy numbers.

Case of Uncertain Measured Eigensolutions

The extension of the sensitivity method in the case of uncertain measured eigensolutions is proposed. Examination of Eqs. (5–8) shows that experimental data affect only the right-hand side.

In a classic situation, the experimental data are considered as perfect and, after processing, as real quantities. If the imprecision of these data is modeled by fuzzy numbers, the vectors with fuzzy coefficients for the eigenvectors and fuzzy numbers for the eigenvalues must be taken into consideration. To ensure the homogeneity of the difference between the measured fuzzy eigensolutions and the real calculated eigensolutions, the fuzzy representation of ordinary real numbers is used for the latter.[7]

By substituting the fuzzy distance vector in Eq. (5), the following fuzzy linear system is obtained:

$$S \Delta \tilde{\boldsymbol{p}} = \tilde{\boldsymbol{b}} \tag{10}$$

To simulate the imprecision, one can choose a triangular fuzzy number (Fig. 1). The membership function characteristics can be arbitrarily chosen. To make the imprecision model more realistic, we chose a representation[10] accepted by the experimental community. If the superscript (ˆ) indicates the data affected by an imprecision on the order of $p\%$ (Ref. 11), for the eigenvectors

$$\hat{z}_\nu^{(s)} = z_\nu^{(s)}\big(1 + r_i \times \max\big(z_\nu^{(s)}\big) \times p/100\big) \tag{11}$$

and for the eigenvalues

$$\hat{\lambda}_\nu^{(s)} = \lambda_\nu^{(s)}\big(1 + r_i \times p/100\big) \tag{12}$$

where r_i is a random number between -1 and 1.

Empirical findings lead to the following statement of uncertainty with respect to measurement errors: eigenvalues $\pm 1\%$ and modes shapes $\pm 10\%$.

For each component of the eigensolutions, the interval boundaries at the α level $\alpha_k = 0$ are obtained by the following expressions for the eigenvectors:

$$\begin{aligned} \tilde{z}_\nu^{(s)} &= \Big[z_\nu^{(s)}\big(1 + r_i \times \max\big(z_\nu^{(s)}\big) \times p/100\big), \\ &\quad z_\nu^{(s)}\big(1 + (r_i + 1) \times \max\big(z_\nu^{(s)}\big) \times p/100\big)\Big], \qquad r_i \in]-1, 0] \\ \tilde{z}_\nu^{(s)} &= \Big[z_\nu^{(s)}\big(1 + (r_i - 1) \times \max\big(z_\nu^{(s)}\big) \times p/100\big), \\ &\quad z_\nu^{(s)}\big(1 + r_i \times \max\big(z_\nu^{(s)}\big) \times p/100\big)\Big], \qquad r_i \in [0, +1[\end{aligned} \tag{13}$$

and for the eigenvalues:

$$\tilde{\lambda}_\nu^{(s)} = \left[\lambda_\nu^{(s)}(1 + r_i \times p/100), \lambda_\nu^{(s)}(1 + (r_i + 1) \times p/100)\right] \qquad r_i \in]-1, 0]$$
$$\tilde{\lambda}_\nu^{(s)} = \left[\lambda_\nu^{(s)}(1 + (r_i - 1) \times p/100), \lambda_\nu^{(s)}(1 + r_i \times p/100)\right] \qquad r_i \in [0, +1[\tag{14}$$

For the calculated eigensolutions, the fuzzy representation of ordinary real numbers for the eigenvectors, with $z_{\nu L}^{(m)} = z_{\nu R}^{(m)}$, leads to the following expressions[7]:

$$\tilde{z}_\nu^{(m)} = \left\lfloor z_{\nu L}^{(m)}, z_{\nu R}^{(m)} \right\rfloor \tag{15a}$$

and for the eigenvalues, with $\lambda_{\nu L}^{(m)} = \lambda_{\nu R}^{(m)}$:

$$\tilde{\lambda}_\nu^{(m)} = \left\lfloor \lambda_{\nu L}^{(m)}, \lambda_{\nu R}^{(m)} \right\rfloor \tag{15b}$$

The left and right boundaries of the measured eigensolutions at each α level α_k can be determined, which gives

$$\left(\tilde{z}_{\nu_i}^{(s)}\right)^{\alpha_k} = \left[z_{\nu_i L}^{(s)}, z_{\nu_i R}^{(s)}\right]^{(\alpha_k)}, \qquad i - 1, \ldots, c$$
$$\left(\tilde{\lambda}_\nu^{(s)}\right)^{\alpha_k} = \left[\lambda_{\nu L}^{(s)}, \lambda_{\nu R}^{(s)}\right]^{(\alpha_k)} \tag{16}$$

The eigenvectors difference is expressed by

$$\left(\Delta \tilde{z}_{\nu_i}\right)^{(\alpha_k)} = \left[z_{\nu i L}^{(s)} - z_{\nu i}^{(m)}, z_{\nu i R}^{(s)} - z_{\nu i}^{(m)}\right]^{(\alpha_k)}, \qquad i = 1, \ldots, c \tag{17}$$

Similarly, the difference for the eigenvalues can be written as

$$\left(\Delta \tilde{\lambda}_\nu\right)^{(\alpha_k)} = \left[\lambda_{\nu L}^{(s)} - \lambda_\nu^{(m)}, \lambda_{\nu R}^{(s)} - \lambda_\nu^{(m)}\right]^{(\alpha_k)} \tag{18}$$

By regrouping Eqs. (17) and (18) in Eq. (10), a system of fuzzy linear equations is obtained for each α_k, which calculates the fuzzy mass and stiffness correction parameters. In a developed form, it can be written as

$$\begin{Bmatrix} \dfrac{\partial z_1^{(m)}}{\partial k_1} & \cdots & \cdots & \dfrac{\partial z_1^{(m)}}{\partial m_r} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \dfrac{\partial z_m^{(m)}}{\partial k_1} & \cdots & \cdots & \dfrac{\partial z_m^{(m)}}{\partial m_r} \\ \dfrac{\partial \lambda_1^{(m)}}{\partial k_1} & \cdots & \cdots & \dfrac{\partial \lambda_1^{(m)}}{\partial m_r} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \dfrac{\partial \lambda_m^{(m)}}{\partial k_1} & \cdots & \cdots & \dfrac{\partial \lambda_m^{(m)}}{\partial m_r} \end{Bmatrix} \begin{Bmatrix} [\Delta k_{1L}, \Delta k_{1R}] \\ \vdots \\ \vdots \\ [\Delta k_{rL}, \Delta k_{rR}] \\ [\Delta m_{1L}, \Delta m_{1R}] \\ \vdots \\ \vdots \\ [\Delta m_{rL}, \Delta m_{rR}] \end{Bmatrix}^{(\alpha_k)}$$
$$= \begin{Bmatrix} \left[z_{1L}^{(s)} - z_1^{(m)}, z_{1R}^{(s)} - z_1^{(m)}\right] \\ \vdots \\ \vdots \\ \left[z_{mL}^{(s)} - z_m^{(m)}, z_{mR}^{(s)} - z_m^{(m)}\right] \\ \left[\lambda_{1L}^{(s)} - \lambda_1^{(m)}, \lambda_{1R}^{(s)} - \lambda_1^{(m)}\right] \\ \vdots \\ \vdots \\ \left[\lambda_{mL}^{(s)} - \lambda_m^{(m)}, \lambda_{mR}^{(s)} - \lambda_m^{(m)}\right] \end{Bmatrix}^{(\alpha_k)} \tag{19}$$

The updating problem when faced with imprecision in the experimental data is to solve system (19) for each α cut α_k. In practice, the solution of Eq. (19) is carried out after the introduction of dimensionless quantities.[7]

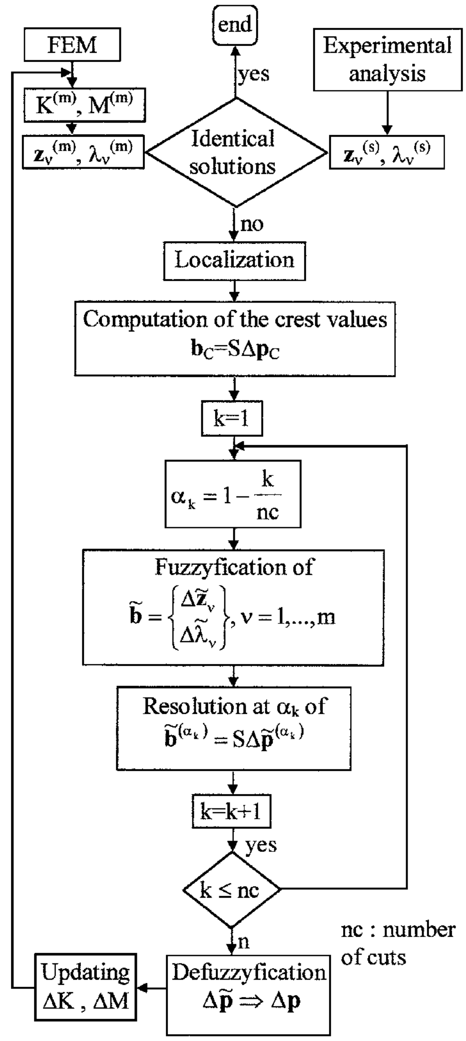

Fig. 2 Modal updating flowchart that takes the imprecise experimental data into account.

Concerning the calculation time, the proposed method must solve nc linear systems with interval coefficients and can be estimated by $(2 \times nc + 1)$ times a classical resolution, due to the technique employed.

This methodology, summarized by the flow chart in Fig. 2, then leads to the determination of fuzzy correction parameters. It is this imprecision, contained in the correction parameters, that will be exploited to elaborate a protocol of dominant modeling errors localization.

Dominant Modeling Errors Localization

In this section, an approach based on the principle of the best subspace method[6,9] is proposed. In our case, we exploit the imprecision on the correction parameters obtained by the proposed formulation.

This methodology exploits the surface occupied by a fuzzy number that represents a correction parameter. Indeed, if a subdomain is sensitive to the imprecision contained in the fuzzy distance vector $\tilde{\boldsymbol{b}}$, the associated correction parameter will be represented by a fuzzy number with significant left and right spreads. The calculated surface for this fuzzy number will be equally important. Inversely, a subdomain less sensitive to imprecision in experimental data will give a fuzzy correction parameter with a smaller surface area. Thus, the localization procedure consists of selecting those parameters associated with the subdomains that present the biggest surface areas.

The algorithm is as follows. The average is calculated for each column of the S matrix. The maximum surface gives the retained parameter and then the first column of the S_d matrix. In the following iterations, the previous columns are retained in S_d and the same calculus is computed with each of the other columns.

The localization procedure stops when the surface of the fuzzy residual error (20) is below a given tolerance:

$$\text{surf}_d^{\text{avg}} = \frac{\sum_{j=1}^{n} \text{surface}(\tilde{\boldsymbol{b}}_j - (S_d \Delta \tilde{\boldsymbol{p}}_d)_j)}{n} \tag{20}$$

In Eq. (20), n represents the number of elements composing the vector $\tilde{\boldsymbol{b}}$, S_d is a submatrix formed of the d selected columns of the S matrix. The solution vector $\Delta \tilde{\boldsymbol{p}}_d$ is evaluated by the solution of the system of fuzzy linear equations $\tilde{\boldsymbol{b}} = S_d \Delta \tilde{\boldsymbol{p}}_d$.

Application to a Test Case

The dominant modeling errors localization approach proposed is illustrated and evaluated by application to a test case. Computer programs were elaborated in MATLAB™ language, and tests were performed on a Hewlett Packard workstation series 700.

Description of the Test Structure

The Group for Aeronautical Research and Technology in Europe (GARTEUR) structure in Fig. 3 is composed of 74 nodes for a total of 222 degrees of freedom and 78 beam elements.

The mechanical and geometrical characteristics of the steel beams are moment of inertia $I = 0.756 \times 10^{-1}$ m^4, cross-sectional area of vertical elements $S_1 = 0.6 \times 10^{-2}$ m^2, cross-sectional area of horizontal elements $S_2 = 0.4 \times 10^{-2}$ m^2, and cross-sectional area of diagonal elements $S_3 = 0.3 \times 10^{-2}$ m^2.

The test case consists of a strictly numerical simulation in which the parameters E and ρ are assumed constant. To separate the section and inertia contributions of the subdomain, each element of the finite element model is modeled by two parallel beams (a) and (b). The mechanical characteristics assigned to each of these beams are defined in the following scheme:

$$E_i, I_i, \rho_i, S_i \equiv \begin{array}{lll} E_i, I_i = 0, \rho_i, S_i & \text{(a)} & K^{(a)}, M^{(a)} \\ E_i, I_i, \rho_i, S_i = 0 & \text{(b)} & K^{(b)}, M^{(b)} \end{array}$$

To simulate the experiment, the terms of section and inertia of the initial model are perturbed. These perturbations concern 15 of the 156 elements of the model (Fig. 4). Experimental eigensolutions are then evaluated using MATLAB software[12] (elements beam1).

The correlation of the eigenvectors is achieved from modal assurance criterion (MAC),[13] whose formulation is given by

$$\text{MAC}\left(z_\nu^{(s)}, z_\nu^{(m)}\right) = \frac{\left|z_\nu^{(s)T} z_\sigma^{(m)}\right|^2}{\left\|z_\nu^{(s)}\right\| \left\|z_\sigma^{(m)}\right\|} \times 100$$

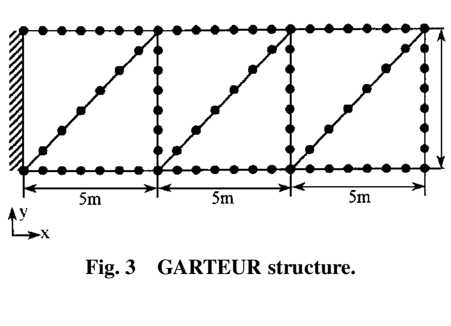

Fig. 3 GARTEUR structure.

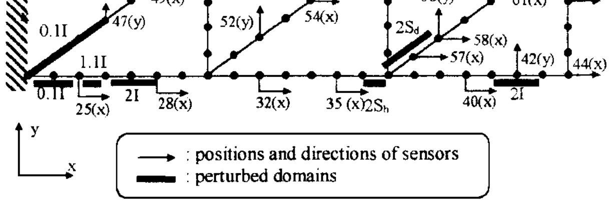

Fig. 4 Perturbations, positions, and directions of the sensors.

The MAC identifies the correlation errors between the experimental eigensolutions $f_\nu^{(s)}$; $z_\nu^{(s)}$ and the analytical eigensolutions $f_\nu^{(m)}$; $z_\nu^{(m)}$, given by, for eigenfrequency error,

$$\text{Err}_f\left(f_\nu^{(s)}, f_\sigma^{(m)}\right) = \frac{\left|f_\nu^{(s)} - f_\sigma^{(m)}\right|}{f_\nu^{(s)}} \times 100$$

for eigenvector (ev) error,

$$\text{Err}_{\text{ev}}\left(z_\nu^{(s)}, z_\sigma^{(m)}\right) = \frac{\left\|z_\nu^{(s)} - z_\sigma^{(m)}\right\|}{\left\|z_\nu^{(s)}\right\|} \times 100$$

and, for eigenvector norm error,

$$\text{Errn}_{\text{ev}}\left(z_\nu^{(s)}, z_\sigma^{(m)}\right) = \frac{\left\|z_\nu^{(s)}\right\| - \left\|z_\sigma^{(m)}\right\|}{\left\|z_\nu^{(s)}\right\|} \times 100$$

Values calculated with MAC are shown in Table 1. This calculation concerns the first 10 eigenvectors of the model and the structure. In Table 1, only the values of MAC greater than 0.60 are shown. Table 2, compares the experimental and finite element results.

The examination of the MAC in Table 2 shows bad correlation of modes 3, 4, and 6. To avoid incoherence and numerical problems, these three modes are not used in the updating procedure. This operation is widespread and even recommended with the updating method used.

Figure 5 shows the comparison between the frequency response functions (FRF) of the structure and the initial model, respectively, for the sensor (node 12, direction x). These FRF where calculated using a proportional damping with a damping ratio of 1%.

Structure Parameterization

The definition of the correction parameters is one of the major problems before the parametric localization procedure. This parameterization is often made from the engineers' point of view, i.e., on the basis of the uncertainties that they consider most important in the model.

The structure parameterization is envisaged here under the following particular aspect: Only the cross-sectional area S and the moment of inertia I are perturbed. These parameters are common to all of the elements composing a subdomain. Figure 6 shows the adopted subdomains division: diagonal and horizontal bars were partitioned into three subdomains and vertical bars into two subdomains.

Two independent parameters S and I are associated to each subdomain. The total number of unknowns is, therefore, $33 \times 2 = 66$ unknowns. Among the 33 subdomains, eight contain perturbed regions.

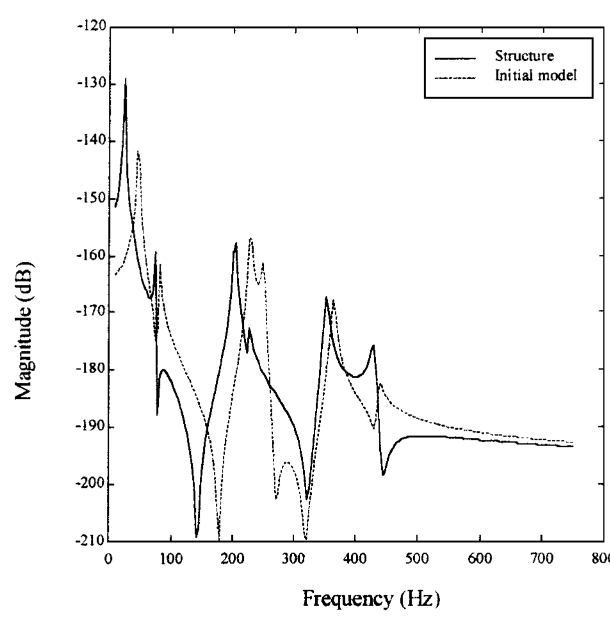

Fig. 5 FRF of the experimental and initial models for the sensor (12x).

Table 1 Values of MAC for the initial model: finite element model

Modes	1	2	3	4	5	6	7	8	9	10
1	98.63	—	—	—	—	—	—	—	—	—
2	—	99.77	—	—	—	—	—	—	—	—
3	—	—	—	89.78	—	—	—	—	—	—
4	—	—	76.63	—	—	—	—	—	—	—
5	—	—	—	—	96.23	—	—	—	—	—
6	—	—	—	—	—	—	66.03	—	—	—
7	—	—	—	—	—	85.24	—	—	—	—
8	—	—	—	—	—	—	—	68.95	—	—
9	—	—	—	—	—	—	—	—	75.64	—

Table 2 Correlation between the initial model and structure

Modes	$f_v^{(s)}$, Hz	$f_\sigma^{(m)}$, Hz	Err_f,%	Errn_{ev},%	Err_{ev},%
1-1	24.17	45.15	86.80	3.60	12.45
2-2	73.31	79.05	7.83	2.94	5.63
3-4	201.95	249.69	23.63	5.16	127.80
4-3	224.54	227.18	1.18	2.08	105.22
5-5	351.84	363.56	3.33	1.88	19.76
6-7	430.30	445.91	3.63	6.29	130.07
7-6	443.76	437.73	1.36	6.54	104.89
8-8	478.30	469.21	1.90	8.16	61.11
9-9	492.45	488.88	0.72	9.70	54.33

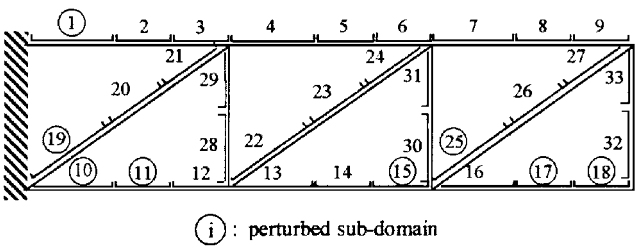

Fig. 6 Partition of the GARTEUR structure into subdomains.

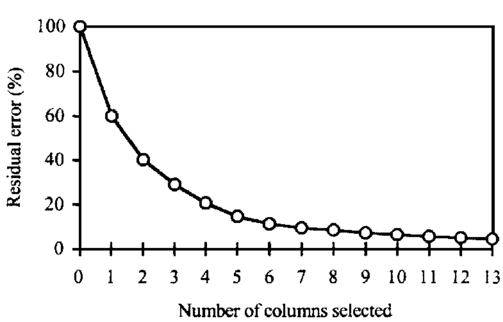

Fig. 7 Evolution of the residual error ε_d: traditional approach.

Application of the Traditional Approach to Localize the Subdomains Presenting Dominant Modeling Errors

This section presents the localization results obtained using the traditional approach.[7] In this application a tolerance is imposed, tol = 1%, on the stability of the residual error. Consequently, the selection stops at the iteration i when $\varepsilon_d^{(i-1)} - \varepsilon_d^{(i)} \le 0.01$.

Selected parameters are the columns of indices (underlined indices correspond to perturbed parameters)

$$32, 33, \underline{43}, \underline{34}, 9, \underline{52}, \underline{44}, 37, 28, 8, 35, 45, \text{and } 53$$

The localization that results when using the traditional approach is shown in Fig. 7. The imposed tolerance criterion selects 13 parameters among 66 defined earlier.

The analysis of the results obtained gives rise to the following observations:

1) The residual error is practically stable from the 13th iteration, i.e., 13th selected parameter. At this stage, 4 perturbed parameters are selected.

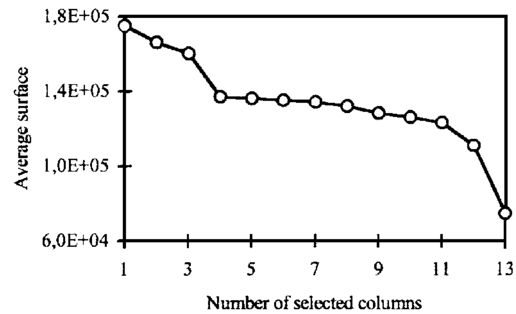

Fig. 8 Evolution of the average surface $\text{Surf}_d^{\text{avg}}$: fuzzy approach.

2) The first selected parameter (number 32) is not a perturbed parameter.

3) The first perturbed parameter is selected from the 3rd iteration.

Application of the Localization Approach with Uncertain Experimental Data (Fuzzy Approach)

The results obtained by the localization approach developed in this paper are presented in this section. It must not be forgotten that, for this approach, the results are analyzed in terms of fuzzy numbers surfaces representing the correction parameters. The selection stops at the iteration i when

$$\text{Surf}_d^{(i-1)\,\text{avg}} - \text{Surf}_d^{(i)\,\text{avg}} \le 0.25 \times \text{Surf}_d^{(1)\,\text{avg}}$$

where $\text{Surf}_d^{(1)\,\text{avg}}$ is the average calculated surface corresponding to the first selected parameter. To compare, a tolerance, tol = 25%, was chosen on the stability of the average surface. This choice means that 13 columns can be selected as in the preceding section.

Selected parameters are the columns of indices (underlined indices correspond to perturbed parameters)

$$\underline{44}, 58, \underline{34}, 39, 40, 59, 54, 47, \underline{43}, 45, \underline{50}, 8, \text{and } 9$$

Figure 8 shows the evolution of the average surface associated to the solutions obtained by the proposed localization approach.

Analysis of the results shows that the proposed approach identifies those parameters that were actually perturbed. Indeed, two of the three first localized parameters were actually perturbed. Note that the first selected parameter is a perturbed parameter (number 44). It can be observed that this localization technique provides, on 13 selected parameters, four are actually perturbed.

With this localization approach, the quality of the results is promising. Nevertheless, the number of localized parameters when using the two approaches is the same.

Considering earlier results, the objective is now to exploit the correction coefficients corresponding to the parameters selected by the two localization approaches. The purpose is to correct the regions presenting dominant modeling errors because it is then possible to obtain analytical eigensolutions similar to those measured on the prototype.

Parametric Updating

For this updating method, it must be remembered that the mass and stiffness matrices of the finite element model are expressed by linear combinations of the mass and stiffness matrices associated

Table 3 MAC matrix for the updated model: traditional approach

Mode	1	2	3	4	5	6	7
1	79.99	——	——	——	——	——	——
2	——	90.93	——	——	——	——	——
3	——	——	——	73.95	——	——	——
4	——	——	71.97	——	——	——	——
5	——	——	——	——	98.64	——	——
6	——	——	——	——	——	——	92.42
7	——	——	——	——	——	91.82	——

Table 4 MAC matrix for the updated model: fuzzy approach

Mode	1	2	3	4	5	6	7
1	99.98	——	——	——	——	——	——
2	——	99.99	——	——	——	——	——
3	——	——	——	99.58	——	——	——
4	——	——	99.63	——	——	——	——
5	——	——	——	——	99.67	——	——
6	——	——	——	——	——	——	92.71
7	——	——	——	——	——	94.47	——

Table 5 Correlation between the experimental and updated eigensolutions: traditional approach

App. modes	Ref. freq.	Calc. freq.	Err_f, %	Err_{ev}, %	$Errn_{ev}$, %
1–1	24.17	29.22	20.91	7.71	7.32
2–2	73.31	70.79	3.44	3.66	4.40
3–4	201.95	218.07	8.01	>50	>50
4–3	224.54	192.38	14.31	>50	45.29
5–5	351.84	338.86	3.69	>50	5.86
6–7	430.30	401.50	6.69	>50	30.13

Table 6 Correlation between the experimental and updated eigensolutions: fuzzy approach

App. modes	Ref. freq.	Calc. freq.	Err_f, %	Err_{ev}, %	$Errn_{ev}$, %
1–1	24.17	27.52	13.86	1.94	1.53
2–2	73.31	74.60	1.76	1.91	1.71
3–4	201.95	223.56	10.69	>50	24.54
4–3	224.54	206.48	8.04	>50	18.54
5–5	351.84	357.68	1.65	11.73	0.55
6–7	430.30	441.92	2.69	>50	11.87

to the subdomains. The coefficients of the linear combination are the correction parameters. If the correction parameters are known, the representative matrices of the updated model can be constructed and the associated eigensolutions can be calculated.

From the parametric localization results, the initial finite element estimation is estimated by taking the first 13 active parameters selected earlier into account.

The modal comparison between the experimental and analytical eigensolutions obtained from the updated models using the traditional approach and the proposed fuzzy approach, respectively, are given in Tables 3 and 4. Given the significant discrepancies between the frequencies of the test structure, the correlation matrix is limited to the terms along the diagonal.

Tables 5 and 6 compare the first six eigenmodes of analytical models updated by the two localization approaches with the experimental data of the real structure.

For comparison purpose, the FRF for the sensor ($12x$), obtained after introduction of the parametric correction (traditional and fuzzy approaches) in the finite element model are shown in Figs. 9 and 10. These FRF are calculated using a proportional damping with a damping ratio of 1%.

To evaluate the enhancements obtained using the two updated analytical models (traditional and proposed localization approaches), Figs. 11 and 12 present for each mode the error on the MAC, i.e., 1-MAC, the smaller the value, the better the correlation, the correlation errors on the frequencies, and the correlation errors on

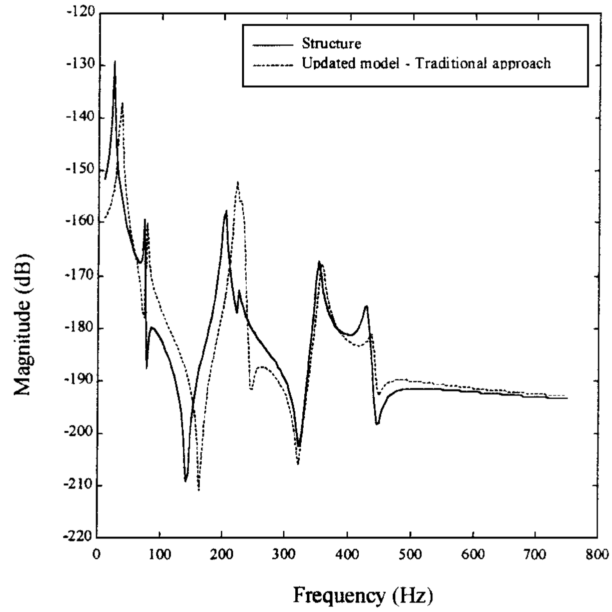

Fig. 9 FRF of the experimental and updated models (traditional approach) for the sensor $12x$.

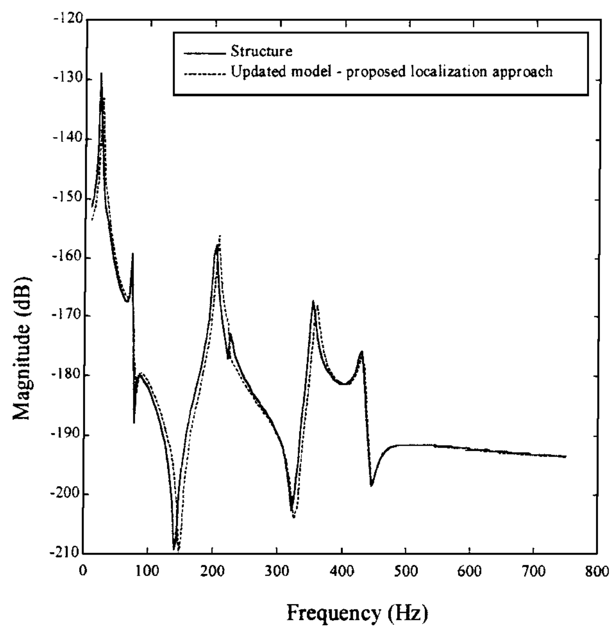

Fig. 10 FRF of the experimental and updated models (proposed localization approach) for the sensor $12x$.

the eigenvectors for the following comparisons: 1) experimental model/analytical model updated using the traditional approach and 2) experimental model/analytical model updated using the fuzzy localization approach. The choice of this representation, with the three comparison criteria per mode, clearly highlights the good and bad correlations.

The MAC coefficients in Tables 3 and 4 show that the modal comparison between the updated base (by the proposed approach) and experimental base gives very good results. Values of MAC are near to 1, the smallest value being 0.93. The correlation is achieved with the first six modes.

The results of Tables 5 and 6 show that for the frequencies, there is a global enhancement of the quality of the results with the proposed approach. The maximal relative discrepancy does not exceed 13.86%, whereas this discrepancy was 86.79% initially. By calculating the mean of gains, the enhancement brought by the updating to the finite element model from the proposed localization approach is 17%. A priori, these eigensolutions are coherent and can be exploited with confidence.

Table 7 Summary of the relative comparison of results

	Granted confidence	
Mode	Traditional approach	Proposed approach
1	Acceptable	Acceptable
2	Acceptable	Good
3	Acceptable but error on the eigenvectors	Acceptable
4	Acceptable but error on the eigenvectors	Acceptable
5	Acceptable	Good
6	Acceptable but error on the eigenvectors	Acceptable

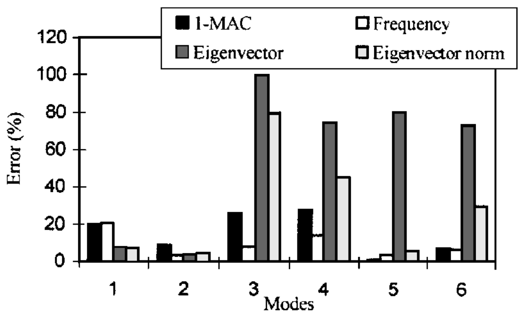

Fig. 11 Relative comparison of the results: traditional approach.

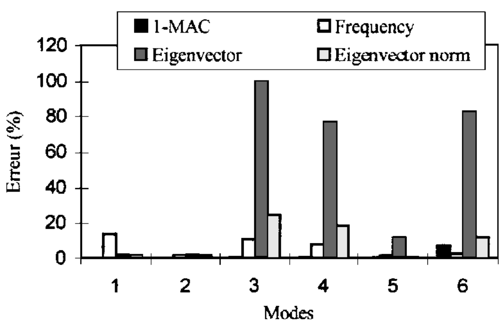

Fig. 12 Relative comparison of the results: proposed fuzzy approach.

For errors on eigenvectors, there are again relatively significant discrepancies for modes 3, 4, and 6. However, for errors on the eigenvectors norms, a good coherence of the eigenvectors is observed.

A summary of the analysis is shown in Table 7, where the confidence granted to the results is evaluated mode by mode.

A good convergence of the analytical model updated by the proposed approach is observed. Moreover, the comparisons of the FRF presented in Figs. 11 and 12 clearly show the enhancement brought by this approach. Nevertheless, note the severe discrepancies on the eigenvectors of modes 3, 4, and 6.

Conclusion

The numerical simulations presented revealed coherent results from two distinct methodologies. The first methodology exploits the evolution of the residual error associated to the parameters or subdomains to localize the regions presenting dominant modeling errors, whereas the second methodology exploits the evolution of the average surface associated to the correction parameters to localize perturbed regions.

The aim is to exploit results from two distinct methodologies used for the same computation. These results are compared with the numerical simulation. The different comparison criteria are the correlation of modes, as well as discrepancies on frequencies and eigenvectors. It is not only a question of evaluating the quality of results in terms of precision but also of judging their credibility by examining the different calculated criteria.

Results obtained by the two methodologies have been compared with the reference data to show the interest in employing one of the two techniques. It has been shown that the traditional localization approach leads to acceptable results. The extension proposed shows that this approach obtains more accurate results.

This test case, therefore, shows that taking uncertainties of experimental data into account in a updating process by the fuzzy formalism contributes to a global result enhancement.

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